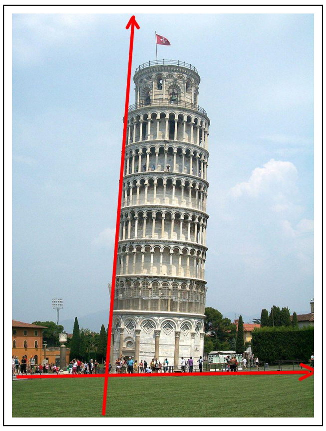
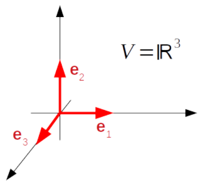
Special Relativity and Classical Field Theory

Lecture and Tutorial – Prof. Dr. Haye Hinrichsen – Maximilian Zemsch – SS 2022



Skew coordinates [Wikimedia]

Exercise 1.1: Skew coordinates (6P)



Consider the Euclidean vector space $V = \mathbb{R}^3$ equipped with the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ and the standard scalar product $g(\mathbf{e}_i, \mathbf{e}_j) = \delta_{ij}$. Define the vectors

$$\begin{aligned} \tilde{\mathbf{e}}_1 &= \mathbf{e}_1 - \mathbf{e}_2 \\ \tilde{\mathbf{e}}_2 &= \mathbf{e}_1 + \mathbf{e}_2 \\ \tilde{\mathbf{e}}_3 &= \mathbf{e}_1 - 2\mathbf{e}_2 - \mathbf{e}_3 \end{aligned}$$

(a) Show that $\{\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2, \tilde{\mathbf{e}}_3\}$ is a basis of V. (1P)

(b) Let $\mathbf{u} = u^i \mathbf{e}_i = \tilde{u}^i \tilde{\mathbf{e}}_i$ be a given vector. Express the new components $\tilde{u}^i$ explicitly in terms of the old components u_i. (1P)

(c) Find the metric tensor $\tilde{g}_{ij}$ in the basis $\{\tilde{\mathbf{e}}_1, \tilde{\mathbf{e}}_2, \tilde{\mathbf{e}}_3\}$. (1P)

(d) Compute the corresponding metric tensor $\tilde{g}^{ij}$ in the dual space. (1P)

(e) Let $\alpha = \mathbf{e}^1 - \mathbf{e}^3$ be a given linear form. Derive an equation for the hyperplane where this form vanishes, that is, find a relation among the components u^i of a vector $\mathbf{u} = u^i \mathbf{e}_i$ in such a way that $\alpha(\mathbf{u}) = 0$. (1P)

(f) Find the representation $\alpha = \tilde{\alpha}_j \tilde{\mathbf{e}}^j$ in the dual basis $\{\tilde{\mathbf{e}}^1, \tilde{\mathbf{e}}^2, \tilde{\mathbf{e}}^3\}$. (1P)

($\Sigma = 6$P)

Please submit your solution as a single pdf file via WueCampus. Deadline is Friday, May 06 at 12:00. Normally we have 12P per sheet. This week we have a warmup exercise with only 6P.